

# Dust Static Spherically Symmetric Solution in $f(R)$ Gravity

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## Abstract

In this paper, we take dust matter and investigate static spherically symmetric solution of the field equations in metric  $f(R)$  gravity. The solution is found with constant Ricci scalar curvature and its energy distribution is evaluated by using Landau-Lifshitz energy-momentum complex. We also discuss the stability condition and constant scalar curvature condition for some specific popular choices of  $f(R)$  models in addition to their energy distribution.

**KEYWORDS:**  $f(R)$  theory, dust solution, constant scalar curvature,  $f(R)$  models, Landau-Lifshitz energy-momentum complex.

**PACS:** 04.50.Kd

## 1 Introduction

Our latest data from different sources, such as Cosmic Microwave Background Radiations (CMBR) and supernova survey indicates that energy composition of the universe is the following: 4% ordinary matter, 20% dark matter and 76% dark energy <sup>1)</sup>. The dark energy has large negative pressure, while the pressure of the dark matter is negligible. The current accelerated expansion of the universe may be due to the presence of dark energy so-called effective cosmological constant. There are various directions aimed to construct

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the acceptable dark energy model. For example, quintessence or phantom models, dark fluid with complicated equation of state, String or M-theory, higher dimensions, more complicated field theories, etc. Despite the number of attempts, there is still no satisfactory explanation about the origin of dark energy.

The  $f(R)$  theory of gravity provides the very natural gravitational alternative for dark energy. The cosmic acceleration can be directly explained by taking any negative power of the curvature (like  $\frac{1}{R}$  term)<sup>2)</sup>. In this way, this theory helps in modification of the model to achieve the consistency with the experimental tests of solar system. A unification of the early time inflation and late time acceleration<sup>3)</sup> is allowed in  $f(R)$  theory. It is also very useful in high energy physics for explaining the hierarchy problem and unification of GUTs with gravity<sup>4)</sup>.

This theory has produced a great number of papers in recent years, for example,<sup>5–7)</sup>. Several features including solar system test<sup>8)</sup>, Newtonian limit<sup>9)</sup>, gravitational stability<sup>10)</sup> and singularity problems<sup>11)</sup> are exhaustively discussed. Much work has been devoted to place constraints on  $f(R)$  models using the observation of CMB anisotropies and galaxy power spectrum<sup>12)</sup>. Kobayashi and Maeda<sup>13)</sup> have studied relativistic stars in this theory gravity. It is shown explicitly that stars with strong gravitational fields develop curvature singularity and hence are prohibited. Erickcek *et al.*<sup>14)</sup> found the unique exterior solution for a stellar object by matching it with interior solution in the presence of matter sources. Kainulainen *et al.*<sup>15)</sup> studied the interior spacetime of stars in Palatini  $f(R)$  gravity.

The vacuum solutions of the field equations in metric  $f(R)$  gravity has attracted many people. Since the spherical symmetry plays a fundamental role in understanding the nature of gravity, most of the solutions are discussed in this context. Multamäki and Vilja<sup>16)</sup> investigated static spherically symmetric vacuum solutions of the field equations. It is shown that solution with constant scalar curvature corresponds to Schwarzschild de Sitter spacetime for a specific choice of constants of integration. Caramês and Bezerra<sup>17)</sup> discussed spherically symmetric vacuum solutions in higher dimensions. Capozziello *et al.*<sup>18)</sup> analyzed spherically symmetric solution using Noether symmetry.

Azadi *et al.*<sup>19)</sup> have studied cylindrically symmetric solutions in Weyl coordinates. They have shown that constant curvature solutions reduce to only one member of the Tian family in General Relativity (GR). Momeni<sup>20)</sup> has found that the exact constant scalar curvature solution in cylindri-

cal symmetry is applicable to the exterior of a string. In a recent work, Sharif and Shamir <sup>21)</sup> have studied exact solutions of Bianchi types *I* and *V* spacetimes in  $f(R)$  theory of gravity. Multamäki and Vilja <sup>22)</sup> investigated non-vacuum solutions by taking perfect fluid. It is found that for a given matter distribution and equation of state, one cannot determine the function form of  $f(R)$ . Hollestein and Lobo <sup>23)</sup> explored exact solutions of the field equations coupled to non-linear electrodynamics.

The energy localization in GR is a serious and long standing issue but without any definite answer. The well-known energy-momentum prescriptions are given by Landau-Lifshitz, Einstein, Bergmann, Papapetrou, Goldberg and Møller. Virbhadra *et al.* <sup>24)</sup> showed that five different energy-momentum complexes yield the same energy distribution for any Kerr-Schild class metric. Chang *et al.* proved that every energy-momentum complex is associated with a Hamiltonian boundary term <sup>25)</sup>. Recently, this problem has been attempted in alternative theories of gravity. In this connection, reasonable amount of work has been published <sup>26)</sup> in teleparallel theory of gravity. Multamäki *et al.* <sup>27)</sup> are the pioneers to discuss energy problem in  $f(R)$  theory of gravity. They presented the concept of energy-momentum complex (EMC) in this theory and generalized the Landau-Lifshitz energy-momentum complex. The prescription is used to evaluate energy-momentum for the Schwarzschild-de-Sitter spacetime. Recently, Sharif and Shamir <sup>28)</sup> found energy densities of plane symmetric and cosmic string spacetimes. They also discussed the stability conditions.

The purpose of this paper is two fold: Firstly, we study non-vacuum static spherically symmetric solutions of the field equations using metric  $f(R)$  gravity in the presence of dust fluid. Secondly, we use generalized Landau-Lifshitz energy-momentum complex to evaluate energy density for constant scalar curvature solution. The paper is organized as follows. In section **2**, we present spherically symmetric field equations and some of the relevant quantities. Section **3** is devoted to study the non-trivial solution of the field equations. In section **4**, we calculate the generalized Landau-Lifshitz energy-momentum complex for constant scalar curvature solution and also discuss some well-known  $f(R)$  models in this context. In the last section **5**, we summarize and discuss the results.

## 2 Field Equations in $f(R)$ Gravity

The action in  $f(R)$  gravity is given by <sup>29)</sup>

$$S = \int d^4x \sqrt{-g} \left[ \frac{f(R)}{2\kappa} + L_M \right], \quad (2.1)$$

where  $f(R)$  is a function of the Ricci scalar and  $L_M$  is the matter Lagrangian depending upon the metric  $g_{\mu\nu}$  and the matter fields. Variation of this action with respect to the metric tensor leads to the following fourth order partial differential equations

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) + g_{\mu\nu} \square F(R) = \kappa T_{\mu\nu}, \quad (2.2)$$

where  $F(R) \equiv df(R)/dR$ ,  $\square \equiv \nabla^\mu \nabla_\mu$  with  $\nabla_\mu$  representing the covariant derivative and  $\kappa (= 8\pi)$  is the coupling constant in gravitational units. Taking trace of the above equation, we obtain

$$F(R)R - 2f(R) + 3\square F(R) = 8\pi T. \quad (2.3)$$

Here  $R$  and  $T$  are related differentially and not algebraically as in GR ( $R = -\kappa T$ ). This indicates that the field equations of  $f(R)$  gravity will admit a larger variety of solutions than does GR. Further,  $T = 0$  does no longer implies  $R = 0$  in this theory.

The Ricci scalar curvature function  $f(R)$  can be expressed in terms of its derivative as follows

$$f(R) = \frac{-8\pi T + F(R)R + 3\square F(R)}{2}. \quad (2.4)$$

This is used to study various aspects of  $f(R)$  gravity, particularly its stability, weak field limit etc. Substituting this value of  $f(R)$  in Eq.(2.2), we obtain

$$\frac{F(R)R - \square F(R) - 8\pi T}{4} = \frac{F(R)R_{\mu\mu} - \nabla_\mu \nabla_\mu F(R) - 8\pi T_{\mu\mu}}{g_{\mu\mu}}. \quad (2.5)$$

In the above equation, the expression on the left hand side is independent of the index  $\mu$ , so the field equations can be written as

$$A_\mu = \frac{F(R)R_{\mu\mu} - \nabla_\mu \nabla_\mu F(R) - 8\pi T_{\mu\mu}}{g_{\mu\mu}}. \quad (2.6)$$

Notice that  $A_\mu$  is not a 4-vector rather just a notation for the traced quantity.

## 2.1 Spherically symmetric spacetime

We take the following static spherically symmetric spacetime

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (2.7)$$

The components of the Ricci tensor are

$$R_{00} = -\frac{1}{4B}[-2A'' + \frac{A'^2}{A} + \frac{A'B'}{B} - 4\frac{A'}{r}], \quad (2.8)$$

$$R_{11} = \frac{1}{4A}[-2A'' + \frac{A'^2}{A} + \frac{A'B'}{B} + 4\frac{B'A}{Br}], \quad (2.9)$$

$$R_{22} = -\frac{1}{2}[\frac{A'r}{AB} - \frac{B'r}{B^2} - 2 + \frac{2}{B}], \quad (2.10)$$

$$R_{33} = \sin^2\theta R_{22}. \quad (2.11)$$

The corresponding Ricci scalar is

$$R = \frac{-2}{r^2B}[1 - B + \frac{r^2A''}{2A} + \frac{A'}{A}(r - \frac{r^2A'}{4A}) - \frac{B'}{B}(r + \frac{r^2A'}{4A})], \quad (2.12)$$

where prime denotes derivative with respect to the radial coordinate  $r$ . The dust energy-momentum tensor is given as

$$T_{\mu\nu} = \rho u_\mu u_\nu, \quad (2.13)$$

where  $u_\mu = \delta_\mu^0$  is the four-velocity in co-moving coordinates and  $\rho$  is the density. Since Eq.(2.5) is independent of index  $\mu$ , so  $A_\mu - A_\nu = 0$  for all  $\mu, \nu$  and yields the following two independent equations

$$-\frac{F''}{B} + \frac{F'}{2B}[\frac{A'}{A} + \frac{B'}{B}] + F[\frac{A'}{BAr} + \frac{B'}{B^2r}] - \frac{8\pi\rho}{A} = 0, \quad (2.14)$$

$$F'[\frac{A'}{2BA} - \frac{1}{Br}] + F[\frac{A''}{2AB} - \frac{A'^2}{4A^2B} - \frac{A'B'}{4AB^2} + \frac{A'}{2ABr} + \frac{B'}{2B^2r} + \frac{1}{r^2} - \frac{1}{Br^2}] - \frac{8\pi\rho}{A} = 0. \quad (2.15)$$

Thus we get a system of two non-linear differential equations with four unknown functions, namely,  $F(r)$ ,  $\rho(r)$ ,  $A(r)$  and  $B(r)$ .

### 3 Solution of the Field Equations

This section is devoted to discuss solution of the field equations by assuming constant scalar curvature which are directly involved in explaining the accelerating universe. In order to take into account acceleration of the present universe, we need to take very small value of the constant to  $f(R)$  <sup>3)</sup>.

The conservation law of energy-momentum tensor,  $T_{\mu;\nu}^\nu = 0$ , for dust matter <sup>22)</sup> gives  $A = \text{constant} = A_0$  (say). Thus the system of field equations (2.14) and (2.15) is reduced to three unknowns with the following two non-linear differential equations

$$\frac{-1}{B}F'' + \frac{B'}{2B^2}F' + \frac{B'}{B^2r}F - \frac{8\pi\rho}{A_0} = 0, \quad (3.16)$$

$$\frac{-1}{Br}F' + [\frac{B'}{2B^2r} + \frac{1}{r^2} - \frac{1}{Br^2}]F - \frac{8\pi\rho}{A_0} = 0. \quad (3.17)$$

Using the assumption of constant scalar curvature ( $R = R_0$ ), i.e.,  $F(R_0) = \text{constant}$ , the field equations become

$$\frac{B'}{B^2r}F(R_0) - \frac{8\pi\rho}{A_0} = 0, \quad (3.18)$$

$$(\frac{B'}{2B^2r} + \frac{1}{r^2} - \frac{1}{Br^2})F(R_0) - \frac{8\pi\rho}{A_0} = 0. \quad (3.19)$$

Now we have two differential equations with two unknowns,  $B(r)$  and  $\rho(r)$ . Subtracting Eq.(3.19) from Eq.(3.18), we have an ordinary differential equation in terms of  $B(r)$  as follows

$$B'r + 2B - 2B^2 = 0 \quad (3.20)$$

which has the following solution

$$B(r) = \frac{1}{1 - c_1 r^2}, \quad (3.21)$$

where  $c_1$  is a constant. Inserting this value of  $B$  in any of the above equations, it follows that

$$\rho = \frac{c_1 A_0 F(R_0)}{4\pi} = \rho_0 \quad (3.22)$$

which is purely a constant. We would like to mention here that if matter density and scalar curvature are constant and also  $\rho_0$  depends on gravitational

constant and effective cosmological constant <sup>30)</sup>, then the field equations become equivalent to the Einstein field equations. The spacetime for constant curvature solution takes the following form

$$ds^2 = A_0 dt^2 - \frac{1}{1 - c_1 r^2} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (3.23)$$

This solution corresponds to the well-known Tolman-Oppenheimer-Volkoff (TOV) spacetime when density is constant and pressure is neglected <sup>31)</sup>. For all ordinary, non-relativistic stars where  $p \ll \rho$  (for example at the center of the sun), we can neglect pressure and can consider an idealized object with a constant density.

The scalar curvature turns out to be  $R_0 = 6c_1$  and hence Eq.(2.3) yields

$$f(R_0) = \frac{1}{2} \left( \frac{-8\pi\rho_0}{A_0} + F(R_0)R_0 \right). \quad (3.24)$$

Substituting the values of  $\rho_0$  and  $R_0$ , it follows that

$$f(R_0) = 2c_1 f'(R_0). \quad (3.25)$$

For the acceptability of any  $f(R)$  model, it is necessary to satisfy this equation. In the next section, we check this condition for some well-known  $f(R)$  models and then calculate energy density using the generalized Landau-Lifshitz energy-momentum complex.

In view of the above information, the universe could start from inflation driven by the effective cosmological constant at the early stage where curvature is very large. With the passage of time, the effective cosmological constant also becomes smaller corresponding to the smaller curvature. After that time, the density of matter or radiations become small and curvature goes to constant value  $R_0$ . Thus expansion could start and cosmological constant can be identified as  $f(R_0)$  in the present accelerating era.

## 4 Landau-Lifshitz Energy-Momentum Complex

Now we evaluate energy density for the constant scalar curvature solution. For this purpose, we use the generalized Landau-Lifshitz energy-momentum

complex. We would like to mention here that this energy-momentum complex is valid only for those solutions which have constant scalar curvature.

The generalized Landau-Lifshitz energy-momentum complex <sup>27)</sup> is given as follows

$$\tau^{\mu\nu} = f'(R_0)\tau_{LL}^{\mu\nu} + \frac{1}{6\kappa}(f'(R_0)R_0 - f(R_0))\frac{\partial}{\partial x^\lambda}(g^{\mu\nu}x^\lambda - g^{\mu\lambda}x^\nu). \quad (4.26)$$

Here  $\tau_{LL}^{\mu\nu}$  is the Landau-Lifshitz energy-momentum complex evaluated in the framework of GR and is given as

$$\tau_{LL}^{\mu\nu} = (-g)(T^{\mu\nu} + t_{LL}^{\mu\nu}), \quad (4.27)$$

where  $t_{LL}^{\mu\nu}$  can be obtained by the following formula

$$\begin{aligned} t_{LL}^{\mu\nu} = & \frac{1}{2\kappa}[(2\Gamma_{\alpha\beta}^\gamma\Gamma_{\gamma\delta}^\delta - \Gamma_{\alpha\delta}^\gamma\Gamma_{\beta\gamma}^\delta - \Gamma_{\alpha\gamma}^\gamma\Gamma_{\beta\delta}^\delta)(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\nu}g^{\alpha\beta}) \\ & + g^{\mu\alpha}g^{\beta\gamma}(\Gamma_{\alpha\delta}^\nu\Gamma_{\beta\gamma}^\delta + \Gamma_{\beta\gamma}^\nu\Gamma_{\alpha\delta}^\delta - \Gamma_{\gamma\delta}^\nu\Gamma_{\alpha\beta}^\delta - \Gamma_{\alpha\beta}^\nu\Gamma_{\gamma\delta}^\delta) \\ & + g^{\nu\alpha}g^{\beta\gamma}(\Gamma_{\alpha\delta}^\mu\Gamma_{\beta\gamma}^\delta + \Gamma_{\beta\gamma}^\mu\Gamma_{\alpha\delta}^\delta - \Gamma_{\gamma\delta}^\mu\Gamma_{\alpha\beta}^\delta - \Gamma_{\alpha\beta}^\mu\Gamma_{\gamma\delta}^\delta) \\ & + g^{\alpha\beta}g^{\gamma\delta}(\Gamma_{\alpha\gamma}^\mu\Gamma_{\beta\delta}^\nu - \Gamma_{\alpha\beta}^\mu\Gamma_{\gamma\delta}^\nu)]. \end{aligned} \quad (4.28)$$

The 00-component of Eq.(4.26) yields

$$\begin{aligned} \tau^{00} &= f'(R_0)\tau_{LL}^{00} + \frac{1}{6\kappa}(f'(R_0)R_0 - f(R_0))\frac{\partial}{\partial x^\lambda}(g^{00}x^\lambda - g^{0\lambda}x^0) \\ &= f'(R_0)\tau_{LL}^{00} + \frac{1}{6\kappa}(f'(R_0)R_0 - f(R_0))\left(\frac{\partial g^{00}}{\partial x^\lambda}x^\lambda + 3g^{00}\right) \end{aligned} \quad (4.29)$$

with

$$\tau_{LL}^{00} = (-g)(T^{00} + t_{LL}^{00}). \quad (4.30)$$

We can find  $t_{LL}^{00}$  from Eq.(4.28) as follows

$$\begin{aligned} t_{LL}^{00} = & \frac{1}{2\kappa}[(2\Gamma_{\alpha\beta}^\gamma\Gamma_{\gamma\delta}^\delta - \Gamma_{\alpha\delta}^\gamma\Gamma_{\beta\gamma}^\delta - \Gamma_{\alpha\gamma}^\gamma\Gamma_{\beta\delta}^\delta)(g^{0\alpha}g^{0\beta} - g^{00}g^{\alpha\beta}) \\ & + g^{0\alpha}g^{\beta\gamma}(\Gamma_{\alpha\delta}^0\Gamma_{\beta\gamma}^\delta + \Gamma_{\beta\gamma}^0\Gamma_{\alpha\delta}^\delta - \Gamma_{\gamma\delta}^0\Gamma_{\alpha\beta}^\delta - \Gamma_{\alpha\beta}^0\Gamma_{\gamma\delta}^\delta) \\ & + g^{0\alpha}g^{\beta\gamma}(\Gamma_{\alpha\delta}^0\Gamma_{\beta\gamma}^\delta + \Gamma_{\beta\gamma}^0\Gamma_{\alpha\delta}^\delta - \Gamma_{\gamma\delta}^0\Gamma_{\alpha\beta}^\delta - \Gamma_{\alpha\beta}^0\Gamma_{\gamma\delta}^\delta) \\ & + g^{\alpha\beta}g^{\gamma\delta}(\Gamma_{\alpha\gamma}^0\Gamma_{\beta\delta}^0 - \Gamma_{\alpha\beta}^0\Gamma_{\gamma\delta}^0)], \quad \alpha, \beta, \gamma, \delta = 0, 1, 2, 3. \end{aligned}$$



which finally gives

$$\begin{aligned}
t_{LL}^{00} &= \frac{1}{16\pi A_0} [(1 - c_1 r^2) \{ \frac{2c_1}{1 - c_1 r^2} + \frac{rc_1}{1 - c_1 r^2} \cot \theta - 2r^2 + \frac{1}{r} \cot \theta \} \\
&+ \frac{1}{r^2} \{ \frac{2c_1}{1 - c_1 r^2} - \frac{1}{r} \cot \theta - 2(1 - c_1 r^2) - 2 \cot^2 \theta \} \\
&+ \frac{1}{r^2 \sin^2 \theta} \{ \frac{2c_1}{1 - c_1 r^2} + \frac{4}{r^2} + \frac{6}{r} \cot \theta + \frac{2c_1}{1 - c_1 r^2} \cot \theta \\
&+ 2 \cot^2 \theta - 2 \cos^2 \theta \}]. \tag{4.31}
\end{aligned}$$

Inserting this value in Eq.(4.30) and then substituting the resulting equation in Eq.(4.29), it follows that

$$\begin{aligned}
\tau^{00} &= f'(R_0) \frac{A_0 r^4 \sin^2 \theta}{1 - c_1 r^2} [ \frac{\rho}{A_0^2} + \frac{1}{16\pi A_0} \{ (1 - c_1 r^2) \{ \frac{2c_1}{1 - c_1 r^2} \\
&+ \frac{rc_1}{1 - c_1 r^2} \cot \theta - 2r^2 + \frac{1}{r} \cot \theta \} + \frac{1}{r^2} \{ \frac{2c_1}{1 - c_1 r^2} - \frac{1}{r} \cot \theta \\
&- 2(1 - c_1 r^2) - 2 \cot^2 \theta \} + \frac{1}{r^2 \sin^2 \theta} \{ \frac{2c_1}{1 - c_1 r^2} + \frac{4}{r^2} + \frac{6}{r} \cot \theta \\
&+ \frac{2c_1}{1 - c_1 r^2} \cot \theta + 2 \cot^2 \theta - 2 \cos^2 \theta \} \} ] + \frac{1}{16\pi A_0} (f'(R_0) R_0 \\
&- f(R_0)). \tag{4.32}
\end{aligned}$$

This is the energy density satisfying the condition of constant scalar curvature. We can evaluate this quantity for different well-known  $f(R)$  models. Also, we check the validity and the stability condition for these models in the context of cosmology.

## 4.1 Energy Density of the First Model

First of all we evaluate energy density for the model  $f(R) = R + \epsilon R^2$ , where  $\epsilon$  is any positive real number. The stability criteria for this model is restricted to  $\epsilon < 0$  which corresponds to  $f''(R) > 0$ . For  $\epsilon = 0$ , GR is recovered in which black holes are stable classically but not quantum mechanically due to Hawking radiations. Since such features also found in  $f(R)$  gravity, hence the classical stability condition for the Schwarzschild black hole can be enunciated as  $f''(R) > 0$ . It is interesting to mention here that this model satisfies the condition of constant scalar curvature for a specific value of the

constant  $c_1$ , i.e.,  $c_1 = \frac{1}{3\epsilon}$ . Further, the stability condition for this model <sup>32)</sup>,  $\frac{1}{\epsilon(1+2\epsilon R_0)} = \frac{1}{5\epsilon} > 0$ , is also satisfied.

The 00-component of the generalized EMC takes the form

$$\begin{aligned}\tau^{00} = & (1 + 12\epsilon c_1) \frac{A_0 r^4 \sin^2 \theta}{1 - c_1 r^2} \left[ \frac{\rho}{A_0^2} + \frac{1}{16\pi A_0} \{ (1 - c_1 r^2) \right. \\ & \times \left\{ \frac{2c_1}{1 - c_1 r^2} + \frac{rc_1}{1 - c_1 r^2} \cot \theta - 2r^2 + \frac{1}{r} \cot \theta \right\} + \frac{1}{r^2} \left\{ \frac{2c_1}{1 - c_1 r^2} \right. \\ & - \frac{1}{r} \cot \theta - 2(1 - c_1 r^2) - 2 \cot^2 \theta \} + \frac{1}{r^2 \sin^2 \theta} \left\{ \frac{2c_1}{1 - c_1 r^2} + \frac{4}{r^2} \right. \\ & \left. \left. \left. + \frac{6}{r} \cot \theta + \frac{2c_1}{1 - c_1 r^2} \cot \theta + 2 \cot^2 \theta - 2 \cos^2 \theta \right\} \right\} \right] + \frac{9\epsilon c_1^2}{4\pi A_0}.\end{aligned}$$

## 4.2 Energy Density of the Second Model

Here we use the model  $f(R) = R - \frac{a}{R} - bR^2$  to evaluate energy density, where  $a$  and  $b$  are any real numbers. This model involves the negative power of the curvature which supports the cosmic acceleration. In this way, any negative power of the curvature can be taken into account to achieve the consistency with experimental tests of Newtonian gravity. However, the model involving such term might not satisfy the stability conditions which can be significantly improved by adding square terms of the scalar curvature. For  $R = R_0 = 6c_1$ , we have

$$f(R_0) = 6c_1 - \frac{a}{6c_1} - 36bc_1^2, \quad f'(R_0) = 1 + \frac{a}{36c_1^2} - 12bc_1. \quad (4.33)$$

The constant curvature condition implies that both the parameters  $a$  and  $b$  are related by the following expression

$$1 - \frac{a}{18c_1^2} - 3bc_1 = 0. \quad (4.34)$$

Further, the stability condition <sup>27)</sup>  $f''(R_0) > 0$  yields  $a + b(R_0)^3 \geq 0$  which is satisfied for  $c_1^2 \geq \frac{5a}{72}$ . Thus the model is acceptable for such choice of  $c_1$ .

Consequently, the energy density takes the form

$$\begin{aligned}
\tau^{00} = & (1 + \frac{a}{36c_1^2} - 12bc_1) \frac{A_0 r^4 \sin^2 \theta}{1 - c_1 r^2} \left[ \frac{\rho}{A_0^2} + \frac{1}{16\pi A_0} \{ (1 - c_1 r^2) \right. \\
& \left. \{ \frac{2c_1}{1 - c_1 r^2} + \frac{rc_1}{1 - c_1 r^2} \cot \theta - 2r^2 + \frac{1}{r} \cot \theta \} \right. \\
& + \frac{1}{r^2} \{ \frac{2c_1}{1 - c_1 r^2} - \frac{1}{r} \cot \theta - 2(1 - c_1 r^2) - 2 \cot^2 \theta \} \\
& + \frac{1}{r^2 \sin^2 \theta} \{ \frac{2c_1}{1 - c_1 r^2} + \frac{4}{r^2} + \frac{6}{r} \cot \theta + \frac{2c_1}{1 - c_1 r^2} \cot \theta \\
& \left. + 2 \cot^2 \theta - 2 \cos^2 \theta \} \} \right] + \frac{1}{16\pi A_0} \left( \frac{a}{3c_1} - 36bc_1^2 \right). \quad (4.35)
\end{aligned}$$

### 4.3 Energy Density of the Third Model

The model considered here is  $f(R) = R - (-1)^{n-1} \frac{a}{R^n} + (-1)^{m-1} b R^m$ , where  $m$  and  $n$  are positive integers and  $a, b$  are real numbers, and is widely used in cosmology. A model of such type with  $m = 1 - \frac{\alpha}{2}$  with  $\alpha$  depending upon the mass of the galaxy is used to approximate galaxies by taking spherically symmetric solutions. It is straightforward that one cannot fit the data for all astronomical masses for a single choice of  $f(R)$  as  $\alpha$  depends upon the mass of individual galaxy. For  $R = R_0$ , the model becomes

$$f(R) = \frac{(6c_1)^{n+1} - (-1)^{n-1}a + (-1)^{m-1}b(6c_1)^{m+n}}{(6c_1)^n}, \quad (4.36)$$

$$f'(R_0) = \frac{(6c_1)^{n+1} + (-1)^{n-1}na + (-1)^{m-1}bm(6c_1)^{m+n}}{(6c_1)^{n+1}}. \quad (4.37)$$

For this form of  $f(R)$ , the constant curvature condition can be written as

$$2(6c_1)^{n+1} - (-1)^{n-1}a(3+n) + (-1)^{m-1}b(3-m)(6c_1)^{m+n} = 0. \quad (4.38)$$

In particular, when  $m = 3$  or  $b = 0$ , we get

$$a = \frac{(-1)^{n-1}2(6c_1)^{n+1}}{n+3}, \quad n \neq 3. \quad (4.39)$$

It is worthwhile to mention here that the stability condition is satisfied for this value of  $a$ .

The corresponding 00-component gives

$$\begin{aligned}
\tau^{00} &= \frac{(6c_1)^{n+1} + (-1)^{n-1}na + (-1)^{m-1}bm(6c_1)^{m+n}}{(6c_1)^{n+1}} \\
&\times \left( \frac{A_0 r^4 \sin^2 \theta}{1 - c_1 r^2} \right) \left[ \frac{\rho}{A_0^2} + \frac{1}{16\pi A_0} \{ (1 - c_1 r^2) \right. \\
&\times \left\{ \frac{2c_1}{1 - c_1 r^2} + \frac{rc_1}{1 - c_1 r^2} \cot \theta - 2r^2 + \frac{1}{r} \cot \theta \right\} \\
&+ \frac{1}{r^2} \left\{ \frac{2c_1}{1 - c_1 r^2} - \frac{1}{r} \cot \theta - 2(1 - c_1 r^2) - 2 \cot^2 \theta \right\} \\
&+ \frac{1}{r^2 \sin^2 \theta} \left\{ \frac{2c_1}{1 - c_1 r^2} + \frac{4}{r^2} + \frac{6}{r} \cot \theta + \frac{2c_1}{1 - c_1 r^2} \cot \theta \right. \\
&+ \left. \left. 2 \cot^2 \theta - 2 \cos^2 \theta \right\} \right] + \frac{1}{16\pi A_0} \\
&\times \frac{(-1)^{n-1}a(n+1) + (-1)^{m-1}b(m-1)(6c_1)^{m+n}}{(6c_1)^n}. \tag{4.40}
\end{aligned}$$

#### 4.4 Energy Density of the Fourth Model

This is an interesting model due to its logarithmic dependence on curvature and it also satisfies the existence of relativistic stars. It is given as follows,  $f(R) = R - a \ln(\frac{|R|}{k}) + (-1)^{m-1}bR^m$ , where  $m$  is a positive integer,  $k$  is a positive real number and  $a$  is any real number. For constant curvature  $R_0$ , we have

$$f(R) = 6c_1 - a \ln\left(\frac{6c_1}{k}\right) + (-1)^{m-1}b(6c_1)^m \tag{4.41}$$

and

$$f'(R_0) = \frac{6c_1 - a + (-1)^{m-1}bm(6c_1)^m}{6c_1}. \tag{4.42}$$

The constant curvature condition is expressed as

$$4c_1 - a \left( \ln\left(\frac{6c_1}{k}\right) - \frac{1}{3} \right) + (-1)^{m-1}b \left( 1 - \frac{m}{3} \right) (6c_1)^m = 0. \tag{4.43}$$

For  $m = 3$  or  $b = 0$ , this reduces to

$$a = \frac{4c_1}{\ln\left(\frac{6c_1}{k}\right) - \frac{1}{3}}. \tag{4.44}$$

The corresponding 00-component is of the form

$$\begin{aligned}
\tau^{00} &= \frac{6c_1 - a + (-1)^{m-1}bm(6c_1)^{m-1}}{6c_1} \\
&\times \left( \frac{A_0 r^4 \sin^2 \theta}{1 - c_1 r^2} \right) \left[ \frac{\rho}{A_0^2} + \frac{1}{16\pi A_0} \{ (1 - c_1 r^2) \right. \\
&\times \left\{ \frac{2c_1}{1 - c_1 r^2} + \frac{rc_1}{1 - c_1 r^2} \cot \theta - 2r^2 + \frac{1}{r} \cot \theta \right\} \\
&+ \frac{1}{r^2} \left\{ \frac{2c_1}{1 - c_1 r^2} - \frac{1}{r} \cot \theta - 2(1 - c_1 r^2) - 2 \cot^2 \theta \right\} \\
&+ \frac{1}{r^2 \sin^2 \theta} \left\{ \frac{2c_1}{1 - c_1 r^2} + \frac{4}{r^2} + \frac{6}{r} \cot \theta + \frac{2c_1}{1 - c_1 r^2} \cot \theta \right. \\
&+ \left. \left. 2 \cot^2 \theta - 2 \cos^2 \theta \right\} \right] + \frac{1}{16\pi A_0} \\
&\times \left\{ a \left( \ln \left( \frac{6c_1}{k} \right) - 1 \right) + (-1)^{m-1} b(m-1)(6c_1)^m \right\}. \tag{4.39}
\end{aligned}$$

## 5 Outlook

This paper investigates solution of static spherically symmetric spacetime with non-trivial matter distribution. We have restricted our analysis to the dust case and obtain solution with assumption of constant scalar curvature. In addition, we have explored energy localization problem using the generalized Landau-Lifshitz energy-momentum complex in  $f(R)$  gravity.

The scalar curvature for this solution turns out to be non-zero constant. This leads to constant density of dust matter and corresponds to the well-known Tolman-Oppenheimer-Volkoff spacetime when density is constant and pressure is neglected<sup>31)</sup>. Thus it is interesting to discuss such solutions for idealized objects where density is constant and pressure can be neglected.

For such solutions with constant curvature, we can use the generalized Landau-Lifshitz energy-momentum complex to discuss energy-momentum distribution. We have evaluated energy density for this solution as well as for certain specific  $f(R)$  models. It is worthwhile to mention here that the well-known  $f(R)$  models satisfy the stability condition as well as constant scalar curvature condition for this solution. The cosmological importance of all these models is also discussed. The model with negative power of the scalar curvature directly supports the cosmic acceleration. If we choose the

model with positive powers of the scalar curvature (higher than 1), the mass of the scalar field can be adjusted to be very large and the stability can be improved. For solutions with zero scalar curvature, the generalized Landau-Lifshitz EMC coincides with Landau-Lifshitz EMC in GR. This work adds some knowledge about the longstanding and crucial problem of the localization of energy. It gives the energy density expressions for important  $f(R)$  models which may help at some stage to overcome the theoretical difficulties in the cosmological and astrophysical context.

The applicability of solutions could be tested by comparing with constraints of the solar system and cosmology. It would be interesting to investigate solutions for non-static spacetimes with energy-momentum tensor of other types of fluids.

### Acknowledgments

We would like to thank the Higher Education Commission, Islamabad, Pakistan for its financial support through the *Indigenous Ph.D. 5000 Fellowship Program Batch-III*. We are also grateful to the Physical Society of Japan for financial support in publication.

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